Beyond Leniency: Why Reputation Costs Matter

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April 23, 2015

Abstract

Our model builds on the analysis of the effects of Leniency programs over collusion, under the assumption of the necessity of communication for cartel. We extend the repeated duopoly model framework of Aubert and al. (2006) by introducing reputation costs, to analyse optimal policy for cartel deterrence by the antitrust authority in the case where costs and evidence are asymmetrically distributed among firms. Indeed, in the case colluding firms are heterogeneous in size and in diversification degree, conviction is likely to have substantial negative reputational effects (customers losses). Substantial reputation loss then directly impact cartel decision, by making firms less prone to report under a leniency program, but also less likely to enter a cartel in first place. Our results show that reputation cost is more efficient for cartel deterrence than a Leniency program, although cartel deterrence from the antitrust authority would not be optimal for social welfare in the case of firms having huge asymmetric reputation costs.

The paper is organized as follows: Section 2 sets up the game. Section 3 analyses the effect of leniency and of harming opponents’ reputation on cartel sustainability. Section 4 considers the optimization of social welfare. Section 5 concludes.

1 Introduction and Literature Review

In the case the antitrust authority has enough resources, Leniency programs are only second best (Motta and Polo, 2003). Indeed, in a repeated monopoly model of symmetric information, a Leniency program that only reduces fines for the first deviating firm would have a deterrence effect when repeat offenders are subject to higher sanctions (Spagnolo, 2003). Aubert et al.(2006) extend these studies by focusing on the consequence of rewards on cartel decision, and find positive rewards have a larger effect than reduced fines for deterrence. Introducing reputation costs into this framework, and considering asymmetries between firms (see Blatter, Emons and Sticher, 2014), we study the comparative effect of negative reputation costs and leniency on cartel deterrence.

2 Collusion game: set-up

We consider both cartel formation and existence rely on communication between the two colluding firms. These communications generate evidence that the antitrust authority can use to punish. The authority can either find this evidence by auditing the firm, or each firm can directly report the evidence.

We start from the simple case where firms have symmetric evidence and cost, to extend to the situation of asymmetric evidence and cost (so that conviction by the authority is not automatic if one firm reports).

2.1 Players

There are two types of players: 2 firms originally colluding and an antitrust authority.

\[ i \in I = \{\text{firms, antitrust authority}\} \]
2.2 Strategy sets

2.2.1 Firms

We start from the case where firms are already under collusive agreements, so they have already chosen between \{ \text{Collude}, \text{Compete} \} as done by Aubert and al. (2006).

Both firms follow a grim-trigger strategy.

The firms’ set of strategies is: \{ \text{Collude}, \text{Deviate} \}, where deviation leads to self-report the collusive agreements to the antitrust authority. If one firm deviates, the other one comes back to competition forever.

- The authority “rewards” the reporting firm by reducing its fine from the leniency $L$. The Leniency both directly decreases the cost of deviating from the cartel agreement and increases the risk of agreement.
- The first mover is the only one benefiting from the Leniency program.

2.2.2 Antitrust authority

The antitrust authority’s set of strategies is:

\{ \text{Audit}, \text{Non-Audit} \}

If no firm reports, the authority always audits and find the cartel with exogenous probability $\rho$:

$$P\{ \text{finding/audit} \} = \rho$$

$$\{ \text{non-finding/audit} \} = 1 - \rho$$

2.3 Payoffs

2.3.1 Firms

The firms attempt to maximize their profits in every period. The firms’ payoff is:

- $\pi^C$: Gross profit if both firms compete (standard Cournot competition)
- $\pi^M$: Gross profit if both firms collude (monopoly industry)
- $\pi^D$: Gross profit if deviation for the firm that compete; the firm not deviating has $\pi^L$

Both firms maximize in each period the expected discounted sum of their profits:

$$\max \sum_{i=0}^{\infty} \delta^i \pi_i$$

The discount rate is the same for both firms: $\delta \in (0,1)$.

We assume: $\pi^C < \pi^M < \pi^D$

The collusion game implies a gain for both firms compared to the competition case, but deviating from the collusion (competing) will further benefit the deviating firm (at the expense of the other). Therefore, the best strategy for firms when collusion is sustainable consists in colluding in every period.
2.3.2 Antitrust Authority

The antitrust authority audits randomly and gives with probability $\rho$ a maximal fine $F$ such that:

$$\pi^M - \rho F > \pi^C$$

The maximum fine $\rho F$ is not large enough to deter collusion, and:

$$\pi^D - L > \pi^M$$

otherwise deviating would never be possible.

The antitrust authority can only impose fines on colluding firms if evidence about collusion is found. In case of asymmetric evidence, the evidence presented by firms does not always meet the requirements set by the antitrust, and reporting does not necessarily lead to conviction if evidence is not sufficient. It maximizes an expected discounted welfare function that is described in section 4.

2.4 Timing

We consider a repeated stage game on infinite horizon.

- $t = -1$: both firms choose to collude and exchange information on cartel via communication.
- In each following period, from $t = 0$ to $\infty$: Firms choose whether they keep on colluding or deviating (hence coming back to competition).

3 Analysis of firms’ behavior and strategies

3.1 Standard Leniency Case

We assume the standard leniency case introduced by Aubert et al. (2006) when antitrust authority relies both on audit and leniency program to deter collusion. We assume that the leniency fine $L$ is inferior to the fine from random audits by the Authority.

Under these conditions, a firm chooses to deviate and self-report to the government authority whenever:

$$\pi^D - L + \frac{\delta}{1 - \delta} \pi^C \geq \frac{1}{1 - \delta} (\pi^M - \rho F)$$

which is equivalent to:

$$(1 - \delta)(\pi^D - L) + \delta \pi^C \geq \pi^M - \rho F$$

In other words, the cartel is stable if:

$$\delta \geq \frac{\pi^D - \pi^M - L + \rho F}{\pi^D - \pi^C - L} \quad (1)$$

Since $\pi^M - \pi^C \geq \rho F$, there is no $\delta$ such that $\delta \geq 1$. The couples $(\rho, F)$ that satisfy this inequality, and thus make collusion sustainable are such that:

$$\delta \leq 1$$

$$\Leftrightarrow \rho \leq \frac{\pi^M - \pi^C}{F}$$

As we can see, the width of the collusion region is not impacted by the existence of a Leniency program and of a Leniency fine. Thus, the stability of the cartel in each period only depends on the possibility of avoiding fine $F$ from random audits for the deviating firm. The only condition for efficiency of the leniency program is then that the expected fine $\rho F$ is large enough, so that the collusion would already be weak without any leniency.
3.2 Harm ing Opponent’s reputation

Our aim is to extend the framework introduced by Aubert et al. (2006) to analyze optimal cartel deterrence. We argue that a firm is more likely to self-report if this involves harming his competitors’ reputation rather than because of a Leniency policy.

Our key assumption is that when a firm deviates and self-reports to authority, his competitors are denounced as being members of a cartel and suffers from a stigma, which appears as an increase in their marginal costs, which go up from \( c \) to \( c + \epsilon \). It can be interpreted as additional advertising costs to mitigate the poor image customers have about them, or legal expenditures due to class actions from consumers. Moreover, we only consider the cases where \( \epsilon \) is such that both firms are able to stay on the market.

Unlike his competitors, the reporting firm does not incur the extra cost and is thus better off after because his equilibrium quantity and profits are higher after deviation (see Appendix A.2) when the grim-trigger strategy is applied (Firms punish the deviating firm by producing at the Cournot competition level).

The deviating firm’s profits go up from \( \pi^C \) to \( \pi^C' = \pi^C + g(\epsilon) \). We assume that demand is linear in quantities, which allows us to make this decomposition.

\( g(\epsilon) \) is a positive increasing function of \( \epsilon \), the increase in marginal costs and captures the private reward a firm obtains from deviating and harming its competitors, which makes it better off.

This firm, knowing that, is willing to deviate and harm his opponents’ reputation (to be better off) whenever:

\[
\pi^D + \frac{\delta}{1-\delta} (\pi^C + g(\epsilon)) \geq \frac{1}{1-\delta} (\pi^M - \rho F)
\]

This implies that the cartel is stable if:

\[
\delta \geq \frac{\pi^D - \pi^M + \rho F}{\pi^D - \pi^C - g(\epsilon)} \quad (2)
\]

Comparing the discount factor we just found with the previous one, we immediately notice that when firms can benefit from harming their opponents’ reputation, the discount factor is always higher than under the previous case of a standard Leniency program. Indeed:

\[
\frac{\pi^D - \pi^M - L + \rho F}{\pi^D - \pi^C - L} \leq \frac{\pi^D - \pi^M + \rho F}{\pi^D - \pi^C - g(\epsilon)} \quad (3)
\]

\( g(\epsilon) \) is a positive increasing function of \( \epsilon \), it can be remarked that there are more couples \( (\rho, F) \) that make collusion sustainable under the Leniency program than under the case where firms can harm their opponents’ reputation through deviation. As shown in the graph below, the deviation region expands.
Indeed, the region above the curve which is closer to the origin (for $\epsilon = \epsilon_1 > 0$) corresponds to the couples $(\rho, F)$ that make collusion unsustainable. When $\epsilon$ increases, the curve that separates the collusion and deviation region moves closer to the origin, and the deviation region is wider: It moves away from the highest curve (the one corresponding to $\epsilon = 0$ and there is less scope for collusive strategies.

In industries where reputation matters a lot ($\epsilon$ is high enough, i.e $\epsilon \geq g^{-1}(\pi^M - \pi^C)$, see Appendix A.3 for the Proof), the discount factor will always be higher than 1 and no collusive strategy can be implemented as a Subgame Perfect Nash Equilibrium of the game.

3.3 Harming Opponent’s reputation when firms have different evidence about the Cartel’s existence

We extend now our analysis of reputation costs as a driver of cartel deterrence to the case where firms have asymmetric evidence about the formation and the existence of a collusive agreement. There is one firm which has always enough evidence to be granted amnesty from the competition policy authority and one firm which does not have enough evidence to be always trusted by the authority.

With probability $\mu$, this firm is granted amnesty from the authority and benefits from his opponents’ reputation losses, but with probability $1 - \mu$, it is considered as a liar by the authority and the consumers and suffers from a reputation loss (an increase in marginal costs of $\epsilon$) instead of his competitors.

His expected profits after the deviation period (when competitors apply grim trigger strategies and compete à la Cournot) will then depend on $\mu$ and are:

$$E(\pi) = \mu \times \pi^C_i(q_i \mid c_j = c + \epsilon) + (1 - \mu) \times \pi^C_i(q_i \mid c_i = c + \epsilon)$$

When demand is linear, these expected profits can be written as: $\pi^C|^{\mu} = \pi^C + h(\epsilon, \mu)$

$h(\epsilon, \mu)$ is increasing in both $\mu$ and $\epsilon$ and is such that:

$$\lim_{\mu \to 1} h(\epsilon, \mu) = g(\epsilon)$$

As in the previous case, it captures the reward a firm obtains by reporting to the Authority and seeing
its competitors’ cost increasing, weighted by the fact that with probability $1 - \mu$, the reporting firm with few evidence sees its own costs increasing.

As before, a firm deviates and self reports whenever: 
$$\pi^D + \frac{\delta}{1 - \delta} (\pi^C + h(\epsilon, \mu)) \geq \frac{1}{1 - \delta} (\pi^M - \rho F).$$

This implies that the cartel is stable if:
$$\delta \geq \frac{\pi^D - \pi^M + \rho F}{\pi^D - \pi^C - h(\epsilon, \mu)} \quad (4)$$

We know that $\forall \epsilon > 0$ we consider (i.e such that there are still two firms on the market), when $\mu < 1$, $h(\epsilon, \mu)$ is less than $g(\epsilon)$, meaning that the firm that has less evidence about cartel formation is less likely to report, since it is less likely to be trusted.

In Antitrust programs where firms have to present a minimum of evidence to the Antitrust Authority to be believed, such low-evidence firms will not self-report.

Moreover, for low values of $\mu$ (when $\mu \to 0$), the function $h(\epsilon, \mu)$ is negative, suggesting that his expected profits the period after he deviated and self-reported are even lower than the case of a standard Cournot competition:

This intuitively suggests that the grim trigger strategy is even more punitive in this case than what it was before, reinforcing the sustainability of the cartel. In other words, $\forall \delta \in (0, 1)$, collusion can be sustained as a Subgame Perfect Nash Equilibrium of the Game. Therefore, it is always true that condition (2) is more constraining than condition (4).

For the firm which has low evidence, $\forall \epsilon > 0$ we consider, there will always be more couples $(\rho, F)$ such that collusion is sustainable than under the previous case where both firms have perfect evidence about the cartel’s formation and existence.

3.4 Summary of firms’ behavior in presence of asymmetric costs

Harming its competitors’ reputation is much stronger incentive than Leniency policy and/or random audits: It rewards the deviating firm through the increase in its competitors’ marginal cost. When the expected damage is high, there is no need for the Antitrust Authority to intervene since a firm will always self-report and deviates. Since there is no need for the Authority to Intervene for high values of $\epsilon$, it can set $\rho = 0$. Nonetheless, even though being rewarded through harming its competitors’ reputation cost may be optimal for cartel deterrence, it might have, as we will see, detrimental effects for Welfare. It may even suggest that under collusion, social welfare can be higher than through self-reporting, when the image costs are high.

4 Welfare Analysis : The Antitrust Authority’s behavior

After having considered the firms’ decisions, we now extend our model to analyze the optimal strategy of the Antitrust Authority. It will enable us to find the budget and the probability of audit that maximizes the Antitrust Authority’s payoff. We will abstract from the Leniency setup in our welfare analysis since both leniency and audits are tools at the disposal of the Authority to deter cartels, and we will focus on the probability of successful audit $\rho$.

The per-period Antitrust Authority’s payoff function is a Welfare function $W = TS - c(B)$ where $TS$ denotes Total Surplus and $c(B)$ is an increasing and convex function of budget $B$.

The Antitrust Authority plays first, internalizing the firms’ behavior and under which conditions they are likely to deviate. We assume that $\delta \sim U[0, 1]$.

The Antitrust Authority’s probability of successful audit is $\rho(B)$ which is such that:
$$\lim_{B \to 0} \rho(B) = 0 \quad \text{and} \quad \lim_{B \to \infty} \rho(B) = 1$$
Moreover, it is also increasing and concave in \( B \), the Budget. There is a tradeoff between a higher probability of cartel detection and higher budget expenses, and that is the reason why the Authority chooses \( B \) and consequently \( \rho(B) \) that maximizes its expected discounted welfare. The Authority maximizes : \( \sum_{i=0}^{\infty} \beta^i W_i \), where \( \beta \in [0,1] \) denotes its discount factor.

### 4.1 Symmetric Costs : Optimal Probability of Detection

We first consider the case where firms have symmetric costs. The Authority’s expected welfare is :

\[
E[W] = \Pr(\text{No deviation}) \times (W|\text{No Deviation}) + \Pr(\text{Deviation}) \times (W|\text{Deviation}) - c(B)
\]

\[
= (\delta \geq \delta_{\text{threshold}})(1-\beta)(\rho(B)(TS_C) + (1-\rho(B))TS_M) + (\delta < \delta_{\text{threshold}})(TS_D + (\beta \frac{1}{1-\beta})TS_C) - c(B)
\]

\[
= \left[ \frac{\pi_M - \pi_C - \rho(B)}{\pi_D - \pi_C} \right] (1-\beta) [TS_M + D] + \left[ \frac{\pi_D - \pi_M + \rho(B)F}{\pi_D - \pi_C} \right] \beta TS_C - c(B)
\]

\( TS_M, TS_D, TS_C \) respectively denote the Total Surpluses in Collusion (M), after Deviation(D) and Self-Reporting, and finally in Cournot competition (C). We have \( TS_M < TS_D < TS_C \). We define \( D = TS_C - TS_M \) as being the deadweight loss because of the cartel’s existence.

Since \( \pi^D - \pi_C > 0 \), the Authority’s maximization problem is equivalent to :

\[
\max_B E[W] = [\pi_M - \pi_C - \rho(B)F] \times [TS_M + D \rho(B)F] + \rho(B)F \times [(1-\beta)TS_D + \beta TS_C] - c(B) \quad (5)
\]

The first-order condition for this is :

\[
e'(B) = \rho'(B)((\pi_M - \pi_C)(TS_C - TS_M) + F((1-\beta)TS_D + (\beta - \rho(B))TS_C
\]

\[
+ (\rho(B) - 1) TS_M + \rho(B) F (TS_M - TS_C))
\]

Since \( \rho(B) \) is concave and increasing, \(-c(B)\) is concave and decreasing, the expected welfare would be the sum of two concave functions and will reach a maximum on \( \mathbb{R}^+ \).

Choosing \( \rho(B) = 1-e^{-\lambda B} \), a cost function \( C(B) = \frac{B^2}{2} \), and assuming linear demand \( P = a - Q \), a numerical application gives an optimal detection rate of \( \rho(B^*) = 4.52\% \) (cf Appendix B.1).

This detection rate corresponds to a budget \( B^* \) that maximizes the expected welfare function.

### 4.2 Policy Breakthrough

We consider \( \rho(B) \) such that \( \rho(B) = 1-e^{-\lambda B} \) and analyze the effect of an increase of the rate parameter \( \lambda \) to \( \lambda' \geq \lambda \).

This can be interpreted as an increase in the effectiveness of the antitrust policy. Intuitively, we would expect a decrease in the optimal budget allocated to cartel detection, since a more efficient Authority can achieve the same outcome with less resources.

Nonetheless, a numerical application (cf Appendix B.2) yields a much higher budget. This puzzling result can be explained because of the existence of a tradeoff between \( \rho(B) \) and \( c(B) \). With a more effective antitrust policy, the benefits of increasing the probability of audit surpass the costs which have not changed, and that is why the Authority (we can think of it as a Social Planner) prefers to allocate even more budget to cartel detection and thus chooses a higher probability \( \rho(B) \) at the optimum.
From this perspective, a sudden breakthrough in antitrust policy would be followed by a sudden increase in the authority’s budget. For instance, the introduction of Corporate Leniency Policy in 1993 in the United States resulted in a drastic increase in the number of dismantled cartels with a budget roughly similar to what it was in 1992. However, from 1994 onwards, the budget of the US Antitrust Authority (Antitrust Division of the Department of Justice) has increased drastically and permanently.

4.3 Antitrust Authority’s choice in presence of asymmetric costs

We extend our welfare analysis to the case where firms can harm their competitors’ reputation through self-reporting, as analyzed in the previous section. The deviating firm sees its competitors’ marginal costs increasing by a factor $\epsilon$.

As before, since the Antitrust Authority plays first, it internalizes both the firms’ behavior and the conditions under which a firm deviates. The Authority’s expected welfare is:

$$E[W] = \left[ \frac{\pi_M - \pi_C - \rho(B) F - g(\epsilon)}{\pi_D - \pi_C - g(\epsilon)} \right] \left( \frac{1}{1 - \beta} \right) \left[ (1 - \rho(B))TS_M + \rho(B)TS_C'' \right]$$

$$+ \left[ \frac{\pi_D - \pi_M + \rho(B) F}{\pi_D - \pi_C - g(\epsilon)} \right] \left[ TS_D + \left( \frac{\beta}{1 - \beta} \right) TS_C' \right] - c(B)$$

$TS_C'$ denotes the Total Surplus in the period following the deviation period (if the firm has deviated) and takes into account the fact that one firm’s marginal costs have risen. For the values of $\epsilon$ we consider (such that both firms stay on the market), we have $TS_C' < TS_C$.

On the other hand, $TS_C''$ refers to the Total Surplus when both firms see their marginal costs increasing. It corresponds to the situation of a successful audit by the Antitrust Authority. We have, for the values of $\epsilon$ we consider, $TS_C'' < TS_C' < TS_C$.

To analyze the Authority’s optimal choice of $\rho(B)$ in this situation, we have to distinguish two cases:
• When $\epsilon \in [0, \epsilon^+]$, where $\epsilon^+$ is such that $TS_M = TS'_C$, the Antitrust Authority chooses $B$ and $\rho(B)$ that maximizes its discounted expected welfare, in an analogous manner to condition (1).

• Nevertheless, when $\epsilon$, the strength of the reputation shock affecting collusive firms is higher than $\epsilon^+$, then $TSM > TSC'$ and the maximal per-period Total Surplus is the one under Collusion. Cartels are not harmful but are welfare-maximizing in this situation. In other words, for such values of $\epsilon$, the optimal choice of $\rho(B)$ is $\rho(B^{**}) = 0$ and thus $B^{**} = 0$.

In fact, government intervention can only worsen an already bad situation in this setup.

In Industries with substantial reputation harm for detected colluders, harming its competitors’ reputation is a powerful incentive that already makes the cartel unsustainable. However, this is made at the expense of social welfare, which is now lower than what it was under the perfectly symmetric costs case. In such a situation, cartel deterrence is not desirable since cartels maximize total surplus.

5 Conclusive Remarks

Analysing the optimal cartels deterrence strategy, we found that reputation shocks are much more efficient in deterring collusion than the use of a standard Leniency program. Indeed, by introducing reputation costs, harming its competitors’ reputation translates in substantial benefits for the self-reporting firm, which is much better off and sees its profits increasing. Incentives for firms to damage their competitors’ image then decrease the sustainability of the cartel, so that the more the expected damage is high (i.e. in industries where corporate image matters), the less is the need for the Antitrust Authority to intervene. When reputation costs are very substantial, cartel deterrence by the Antitrust Authority is no longer optimal for social welfare: a laissez-faire policy turns out to be welfare improving.

6 References and Bibliography


• Blatter, M., Emons, W., Sticher, S., 2014. Optimal Leniency Programs when Firms have Cumulative and Asymmetric Evidence.


• Harrington, J., 2013. Corporate Leniency Programs when Firms have Private Information: The Push of Prosecution and the Pull of Pre-Emption, Journal of Industrial Economics 61, 1-27.


Appendix A. Firms’ behavior: A numerical analysis

A.1 Trivial Leniency case

Throughout our numerical applications, we will consider a linear demand \( P = a - Q \) as well as constant marginal costs \( c \) so that we can derive the profits under collusion, deviation and competition which are as follows:

\[
\pi^C = \frac{(a - c)^2}{9} \quad \pi^M = \frac{(a - c)^2}{8} \quad \pi^D = \frac{9(a - c)^2}{64}
\]

Using the discount factor condition (1) we see that the set of sustainable collusive strategies satisfies:

\[\rho \leq \frac{(a - c)^2}{72F}. \]

Since \( \rho F \leq \pi^M - \pi^C \), the condition above always holds.

A.2 Reputation costs: A numerical example

If a firm deviates, its competitors’ marginal costs will increase by \( \epsilon \) so that the reporter new profits after deviation (when all firms compete) are now:

\[
\pi'^C = \frac{(a - c + \epsilon)^2}{9} = \frac{(a - c)^2 + \epsilon(2(a - c) + \epsilon)}{9}
\]

Indeed, as we suggested, when demand is linear, we can make the decomposition \( \pi'^C = \pi^C + g(\epsilon) \) where \( g(\epsilon) = \frac{\epsilon(2(a - c) + \epsilon)}{9} \). It is a strictly increasing and positive function of \( \epsilon \). Therefore, the deviating firm sees its profits increasing through harming its competitors’ reputation.

Recall that we only consider the values of \( \epsilon \) for which the two firms stay on the market, so in our case, with \( P = a - Q \), \( \epsilon \in \left[0, \frac{a-c}{2}\right] \). The set of collusive strategies is the set of \( (\rho, F) \) such that condition () holds, and satisfies thus:

\[
\frac{(a - c)^2}{9} + \frac{\epsilon [2(a - c) + \epsilon]}{9} \leq \frac{(a - c)^2}{8} - \rho F
\]

\[
\Leftrightarrow \frac{(a - c)^2}{72} - \frac{72\rho F}{72} \geq \frac{8\epsilon [2(a - c) + \epsilon]}{72}
\]

\[
\Leftrightarrow 72\rho F \leq (a - c)^2 - 16\epsilon(a - c) - 8\epsilon^2(a - c)^2 \left[1 - \frac{16\epsilon}{(a - c)} - \frac{8\epsilon^2}{(a - c)^2}\right]
\]

\[
\Leftrightarrow \rho \leq \frac{(a - c)^2}{72F} \left[1 - \frac{16\epsilon}{(a - c)} - \frac{8\epsilon^2}{(a - c)^2}\right]
\]

As suggested in the core of the paper, this new condition is always less than \( \frac{(a - c)^2}{72F} (g(\epsilon) \) is positive), so that there are less couples \( (\rho, F) \) for which collusion is sustainable. The deviation region is wider \( \forall \epsilon > 0 \) than under a standard leniency program.

A.3 Threshold for which there is no need for the government authority to Audit

If \( \epsilon \) is high enough, \( \delta > 1 \) always and collusion is not a sustainable strategy. That is, if:

\[
\pi^D - \pi^M - \rho F \geq \pi^D - \pi^C - g(\epsilon)
\]

\[
\Leftrightarrow \pi^M - \pi^C \geq \rho F + g(\epsilon)
\]

which implies that, since \( g(\epsilon) \) is strictly increasing:

\[
\epsilon \geq g^{-1}(\pi^M - \pi^C)
\]

For values of \( \epsilon \) higher than this threshold, even without the threat of an audit from the Authority, the private reward a firm obtains by deviating and harming its competitors is high enough to make collusion unsustainable.
A.4 Numerical application when firms have asymmetric evidence

When a firm does not have enough evidence about the cartel’s formation and existence, its expected profits are:

$$\pi^C_{|\mu} = \pi^C + h(\epsilon, \mu)$$

Using a linear demand, this gives:

$$\pi^C_{|\mu} = \frac{(a - c)^2}{9} + \frac{6\mu(a - c) - 4(a - c) + \epsilon}{9}, \text{ so that } h(\epsilon, \mu) = \frac{6\mu(a - c) - 4(a - c) + \epsilon}{9}$$

Using conditions on the discount factor $\delta$, we find that the couples $(\rho, F)$ such that collusion is sustainable satisfy:

$$\frac{(a - c)^2}{9} + \frac{6\mu(a - c) - 4(a - c) + \epsilon}{9} \leq \frac{(a - c)^2}{8} - \rho F$$

$$\Leftrightarrow (a - c)^2 - 72\rho F \geq 8\epsilon [(a - c)(6\mu - 4) + \epsilon]$$

$$\Leftrightarrow (a - c)^2 \geq (48\mu - 32)\epsilon(a - c) + 8\epsilon^2 + 72\rho F$$

$$\Leftrightarrow 72\rho F \leq (a - c)^2 \left[1 - \frac{(48\mu - 32)\epsilon}{(a - c)} - \frac{8\epsilon^2}{(a - c)^2}\right]$$

$$\Leftrightarrow \rho \leq \frac{(a - c)^2}{72F} \left[1 - \frac{(48\mu - 32)\epsilon}{(a - c)} - \frac{8\epsilon^2}{(a - c)^2}\right]$$

For $\mu = 1$, this condition is equivalent to the condition above, where firms have perfect evidence about the cartel’s formation and existence. For $\mu < 1$, this condition is less restrictive than the previous one (recall that condition (2) is more constraining than condition (4)). The firm, threatened by a potential increase of its own costs, is less likely to deviate than when it has perfect evidence about costs.

B Appendix B.: Welfare analysis

B.1 Welfare function

Let us detail the form of the welfare function. We recall:

$$W = TS - c(B)$$

where $TS = CS + PS$ the sum of consumer and producer surplus and $p(B) = (1 - e^{-\lambda B})$ with $B \geq 0$.

The antitrust authority is uncertain of whether the industry she investigates has a sustainable or an unstable collusion. Therefore it maximises expected welfare. The uncertainty comes from the different possible forms of the total surplus depending on the type of the industry.

There are two possible types:

- "No deviation": Either the industry has a sustainable collusion, which means that the discount factor of the industry is above the threshold discount factor.
- "Deviation": Otherwise, the industry has an unstable collusion, which means that the discount factor of the industry is below the threshold discount.

Since the authority does not observe the industry’s type, it assigns a probability to each possibility, taking into account the conditions under which a firm deviates. Thus:

$$E[W] = \Pr(\text{No deviation}) \times (W|\text{No Deviation}) + \Pr(\text{Deviation}) \times (W|\text{Deviation}) - c(B)$$
Let us now turn to each term of this equation.

As mentioned before, the first and third terms are probabilities assigned by the antitrust authority by taking into account the conditions under which a firm deviates. We recall that we choose \( \delta \sim \text{Uniform}[0, 1] \) for convenience, such that:

\[
\Pr(\text{No deviation}) = \Pr(\delta \geq \delta_{\text{threshold}})
\]

We recall that collusion is sustainable whenever:

\[
\delta \geq \frac{\pi_M - \pi_C - \rho F}{\pi_D - \pi_C}
\]

Let us now detail the form of the total surplus expressions, using our linear demand function: \( P = a - Q \).

This implies that we have:

\[
TS_C = \frac{4(a - c)^2}{9}, \quad TS_M = \frac{3(a - c)^2}{8}, \quad \text{and } TS_D = \frac{55(a - c)^2}{128}
\]

where \( TS_C \) is the total surplus in competition, \( TS_M \) is the total surplus in collusion and \( TS_D \) is the total surplus for the deviation period. We normalize \( (a - c)^2 = 1 \) for convenience in our calculations.

A numerical application of the maximization program choosing \( \lambda = 5 \) yields an optimal budget of \( B^* = 0.00926042 \) and an optimal detection rate of \( \rho(B^*) = 4.52\% \).

B.2 Policy breakthrough

When considering an increase of \( \lambda \) to \( \lambda' \geq \lambda \) choosing \( \lambda' = 20 \), the maximisation program now yields and optimal budget of \( B_{2}^* = 0.0187143 \).
We can indeed see that if the antitrust authority is more effective but invests the same budget $B^*$ as before, it has a detection rate of

$$\rho'(B^* = 0.00926042) = 1 - e^{-20B^*} = 8.93\%$$

which is indeed quite higher than our first detection rate.

If now the more effective ($\lambda = 20$) antitrust authority chooses to invest the new optimal budget $B^*_2$, it has a new probability of detection of

$$\rho'(B^*_2 = 0.0187143) = 1 - e^{-20B^*_2} = 31.22\%$$

### B.3 Welfare analysis : Impact of Asymmetric Costs

Under the case both firms are detected by the antitrust authority, they both suffer reputation costs.

Then, the profit for both firms is:

$$\pi'' = \frac{(a - c)^2}{9} + \frac{4c^2}{9} - \frac{4\epsilon(a - c)}{9}$$

Then:

$$TS''_C = \frac{4(a - c)^2}{9} + \frac{11\epsilon^2}{18} - \frac{4\epsilon(a - c)}{9} \text{ and } TS''_C > TS_C$$

$$TS'_C = \frac{4\epsilon^2}{9} + \frac{16\epsilon}{9} - \frac{16\epsilon(a - c)}{9} \text{ and } TS''_C < TS'_C$$

Where $TS'_C$ is the total surplus if one firm deviates and the other firm suffers a stigma $\epsilon$; $TS''_C$ is the new total surplus in case the authority detects the cartel and punishes both firms because none have deviated.

When $\epsilon \in \left[\frac{5}{22}(a - c), \frac{1}{2}(a - c)\right]$ then $TS_M > TS'_C$.

Thus $\epsilon$ will never be superior to $\frac{(a - c)}{2}$ (recall that we consider the cases such that both firms stay
on the market), and under the case $\epsilon \geq \frac{5}{22}$ the optimal $\rho$ chosen is zero. In other words, when the increase in marginal cost due to reputation harm are bigger than $22.73\%$ of $(a - c)$ (here it is 1), then the authority’s prefers not to audit the industry.